

# REPORT ON THE NOS WORKSHOP ON RESIDENCE/ FLUSHING TIMES IN BAYS AND ESTUARIES

Silver Spring, Maryland  
September 2005



**noaa** National Oceanic and Atmospheric Administration

---

U.S. DEPARTMENT OF COMMERCE  
National Ocean Service  
Office of Coast Survey  
Coast Survey Development Laboratory

**Office of Coast Survey  
National Ocean Service  
National Oceanic and Atmospheric Administration  
U.S. Department of Commerce**

The Office of Coast Survey (CS) is the Nation's only official chartmaker. As the oldest United States scientific organization, dating from 1807, this office has a long history. Today it promotes safe navigation by managing the National Oceanic and Atmospheric Administration's (NOAA) nautical chart and oceanographic data collection and information programs.

There are four components of CS:

The Coast Survey Development Laboratory develops new and efficient techniques to accomplish Coast Survey missions and to produce new and improved products and services for the maritime community and other coastal users.

The Marine Chart Division collects marine navigational data to construct and maintain nautical charts, Coast Pilots, and related marine products for the United States.

The Hydrographic Surveys Division directs programs for ship and shore-based hydrographic survey units and conducts general hydrographic survey operations.

The Navigation Services Division is the focal point for Coast Survey customer service activities, concentrating predominantly on charting issues, fast-response hydrographic surveys and Coast Pilot updates.

# REPORT ON THE NOS WORKSHOP ON RESIDENCE/ FLUSHING TIMES IN BAYS AND ESTUARIES

Frank Aikman, III  
Lyon W.J. Lanerolle

September 2005



**noaa** National Oceanic and Atmospheric Administration

---

U.S. DEPARTMENT  
OF COMMERCE  
Carlos Gutierrez, Secretary

Office of Coast Survey  
Captain Roger L. Parsons, NOAA

National Oceanic and  
Atmospheric Administration  
Conrad C. Lautenbacher, Jr.,  
VADM USN (Ret.), Under Secretary

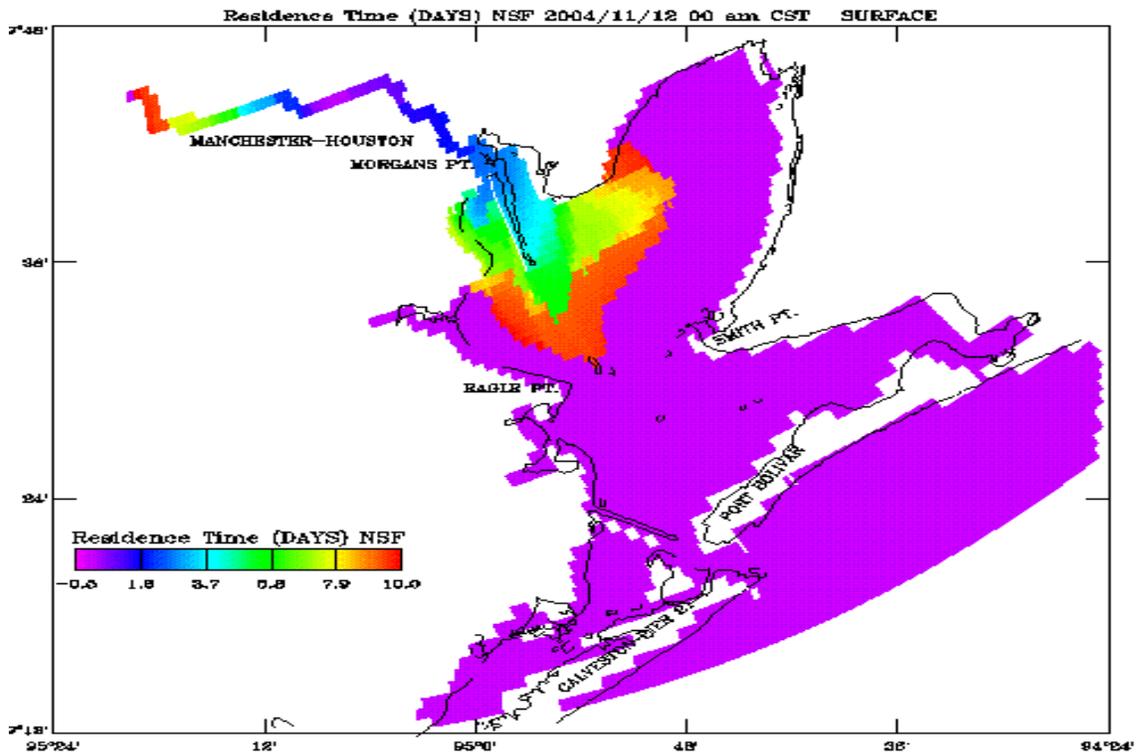
National Ocean Service  
Richard W. Spinrad, Ph.D.  
Assistant Administrator

Coast Survey Development  
Laboratory  
Mary Erickson

## NOTICE

Mention of a commercial company or product does not constitute an endorsement by NOAA. Use for publicity or advertising purposes of information from this publication concerning proprietary products or the tests of such products is not authorized.

# Report on the National Ocean Service Workshop on Residence/Flushing Times in Bays and Estuaries



Frank Aikman III  
Office of Coast Survey, NOAA

Lyon W. J. Lanerolle  
Office of Coast Survey, NOAA

# Table of Contents

Executive Summary .....	iv
1. Introduction.....	1
2. Motivation: Some Background on Residence Time .....	2
3. Workshop Objectives and Organization .....	3
4. Workshop Results and Outcomes .....	4
4a. Applications Session Summary.....	5
4b. Measurements Session Summary .....	6
4c. Algorithms Session Summary .....	6
5. Summary, Recommendations and Future Directions .....	7
Acknowledgements.....	10
References.....	11
Appendix A : Workshop Agenda.....	13
Appendix B : Summaries of the Presentations .....	15
Appendix C : Attendees .....	20

## Executive Summary

Residence times are extremely useful in determining water contamination and nutrient levels, distributions of organisms, and their spatio-temporal variations in bays and estuaries. Therefore, it is important to know if hydrodynamic circulation models could provide higher-resolution estimates of residence times in bays, estuaries and small embayments within them than those available from (simpler) box models and direct measurements. In order to facilitate this, a workshop entitled "NOS Workshop on Residence/Flushing Times in Bays and Estuaries", funded through a NOS Partnership Project, was held at NOAA in Silver Spring, MD, in June 2004. The objectives of the workshop were to investigate: (i) the scope of applications of residence times, (ii) the state of the art methods for the calculations of residence times, (iii) the measurements required to evaluate residence times and (iv) with respect to the use of numerical hydrodynamic models, (a) the various numerical techniques used to determine residence times and (b) comparisons of the Lagrangian (particle tracking) versus Eulerian (tracer patch) approaches to computing residence times.

Thirty-four experts from academic institutions, private industry and government organizations participated in the two day workshop, which consisted of a sequence of invited talks (from the pool of participants) followed by three concurrent break-out sessions on the topics of applications, measurements and algorithms associated with the determination of residence times. Present shortcomings and future directions were also discussed and summaries of these are included in this report.

The conclusions and recommendations resulting from this workshop are: (1) that there is a hierarchy of models that can be used to assess transport time scales and that often the lower resolution models are sufficient; (2) when higher resolution is required hydrodynamic models can be used to compute residence times via (2a) the concentration patch approach, (2b) the Lagrangian particle path approach, (2c) the residual velocity and salinity intrusion method and (2d) the dynamical systems approach (with the use of Synoptic Lagrangian Maps); it is desirable (3) to calibrate (2a) against (2b) with an appropriate form of numerical dispersion included in the latter; (4) to compare the various residence time estimates resulting from the above four approaches ((2a)-(2d)); and (5) to examine residence times in several well known bays and estuaries for which observed data are available.

# 1. Introduction

In 2003, the National Ocean Service's (NOS) National Centers for Coastal Ocean Science (NCCOS) enquired if the hydrodynamic circulation models used by NOS' Coast Survey Development Laboratory (CSDL) could provide spatially and temporally detailed estimates of residence times in bays, estuaries, and small embayments within them. This discussion grew into a proposal to the 2003 NOS Partnership Program, entitled *Flushing and Residence Time Estimation Using NOS Estuarine Hydrodynamic Model-based Forecast Systems*. We hypothesized that the sophisticated 3-dimensional hydrodynamic models that have been developed for many estuaries such as the Galveston Bay Operational Forecast System (GBOFS: see Schmalz, 2004) and the Port of New York/New Jersey Operational Forecast System (NYOFS: see Wei, 2003) would produce more refined estimates of residence time than those based on the traditional models and help better understand the factors that affect it.

Residence time is an important factor in determining levels of contamination and concentrations of nutrients in a water body and their potential effects on estuarine organisms. It is commonly used in producing comparative assessments of how estuaries respond to human use. The calculation of residence time has been predominantly based on empirical studies using tracers (for example, salinity or dye), very simple box models or simple tidal prism models, and 1- and 2-dimensional models (see for example: Sheldon and Alber, 2002; Miller and McPherson, 1991; Hagy et al., 2000; and Signell and Butman, 1992) which have attempted to address some of the weaknesses associated with the simpler box models. We proposed to begin using the 3-dimensional hydrodynamic models that numerically represent the physics that dominate circulation in any particular estuary. Some of the advantages of using hydrodynamic models are: (i) no particular model/behavior for residence times is assumed and a more fundamental approach for calculating them is adopted (for example, using Lagrangian particles – Sections 2, 4c), (ii) due to high spatio-temporal resolution of model outputs, the spatio-temporal behavior of residence times (which is a local quantity – Section 2) can be investigated in domains of arbitrary size, and (iii) several numerical methods can be used with model outputs to calculate and compare residence times (for example, passive tracer dispersion analysis, Lagrangian particle path analysis, residual velocity and salinity intrusion analysis and Synoptic Lagrangian Map analysis – Section 5).

The NOS Partnership proposal was funded and is being used (a) to support a visiting scientist with experience in residence time modeling to work with the NOS scientists, (b) to sponsor a workshop with invited experts to discuss residence time estimation from numerical hydrodynamic models, and (c) to conduct an in situ passive tracer experiment in the Houston Ship Channel with which to evaluate the hydrodynamic model of the GBOFS system and to complete a similar effort previously carried out in New York Harbor (to evaluate the hydrodynamic model of the NYOFS system; see Wei et al., 2004).

This report presents the results of the workshop, (b) above, held on June 8-9, 2004 in Silver Spring, MD.

## 2. Motivation: Some Background on Residence Time

Estuarine ecosystems involve the transport of materials (e.g., nutrients, plants, animals, and suspended particles) in a fluid medium, and the transport phenomena involve a variety of time and space scales. Without breaking transport down to its constitutive processes (advection, diffusion, etc.), simple aspects of it can be determined by looking at some transport time scales which may overlook or amalgamate the underlying physical processes to a great extent. The transport time scales of interest to this workshop are age, residence and flushing time scales and, in particular, the residence time scale. Age is defined as the time a particle/fluid parcel has expended in order to travel to a specific location within a pre-defined region/control volume since entering it through one of its boundaries; therefore, it is unique to each particle/fluid parcel and particles/fluid parcels at different locations within this region have different ages. Residence time is how long a particle/fluid parcel starting at a specific location within this region, will take to leave it through one of its boundaries or alternatively, how long it will remain within this region before exiting it from one of its boundaries. The concepts of age and residence time are complementary to each other so that relative to a common spatial location within a region, age plus residence time equals transit time which is the total time that a particle/fluid parcel spends (between entrance and exit) in this region (Zimmerman, 1976). Flushing time is defined as the time required to reduce the concentration of a tracer by a pre-determined factor (say an e-folding factor) within a given region. Flushing time is usually associated with tracers/dissolved materials (continuous quantities) and residence time and age are associated with both tracers/dissolved materials and particles (discrete quantities). Flushing time is an integrative quantity (thereby ignoring local processes and information) but residence time and age are (spatially) local quantities and are very useful in understanding locally occurring phenomena (Monsen et al., 2002; Sheldon and Alber, 2002). In this respect, the estimation and study of these times in localized regions such as bays and estuaries are particularly relevant. A more in depth discussion of the above mentioned transport time scales can be found in Zimmerman, 1976 and in Tekeoka, 1984.

Residence times are extremely useful in determining, for example, pollutant concentrations, distributions of plankton (Basu and Pick, 1996), harmful algal blooms (Bricelj and Lonsdale, 1997), pelagic bacteria (Painchaud et al., 1996) and dissolved nutrients (Andrews and Muller, 1983) in bays and estuaries, all of which have a significant impact on humans. Some scientific investigators (Boynton et al., 1995) have argued that residence time is a sufficiently significant concept that it could form a basis for the comparative analyses of nutrient budgets in ecosystems (some examples on denitrification are given in Dettmann, 2001 and in Seitzinger, 2000)). Furthermore, due to the spatially local nature of residence times, they are very useful in identifying and quantifying spatially inhomogeneous phenomena and processes in sub-domains of bays, estuaries and ecosystems.

Definitions and calculations of residence times, flushing times and ages do not always use consistent methods, and in most cases the models used for obtaining them (e.g. simple box models) are derived by assuming certain idealized flow conditions (e.g. complete and instantaneous mixing in the region of interest). There are, however,

important physical processes to take into account, such as tides which cause dissolved materials/tracers and particles/fluid parcels to oscillate in and out of bays, estuaries and any sub-domains of interest within them. Simple models, for example the low-order box models, are adequate if the concern is for the average condition in an entire bay or estuary, but they greatly underestimate, or do not address, the residence times associated with smaller regions with restricted circulation, such as urban harbors and embayments. Hence, exploring the different methods of estimating residence times is of paramount importance and was a key theme in this workshop.

Traditionally, residence times have been evaluated using only measured data but in the past couple of decades, with the advent of sophisticated 3-dimensional numerical ocean models, much spatially and temporally extensive flow-field information (for example, water density, temperature, salinity and velocity distributions) can be produced from numerical simulations. These results, with their high spatial and temporal resolutions, can be used to achieve more refined estimates of residence time for the computational regions of application and also to determine the physical factors most affecting their values and spatial distributions. Residence times can be calculated either by an Eulerian approach (tracer/concentration) or by Lagrangian particle path/trajectory information. The former is ideally suited to simulate substance distributions in an entire bay or estuary and the latter for the simulation of the transport of a substance locally in various sub-domains within a bay or estuary. The particle tracking approach is particularly useful in that particles carry localized flow-field information (salinity, temperature, velocities, etc.) while also being subject to the effects of the physical processes (for example, advection, diffusion, etc.). Because particles are discrete, it is easy to determine and examine their statistics (for example, counting them, testing whether they crossed a pre-defined boundary, etc.). In this workshop it was desired to explore these concepts to a greater extent and also discuss other techniques for estimating residence times from hydrodynamic model outputs.

### **3. Workshop Objectives and Organization**

The objectives of the workshop were to discuss and assess: (1) applications of residence/flushing time computations (addressing such concerns as hazardous materials, port development, fish stock assessment, eutrophication, pollution); (2) the state of the art methods for the determination of residence time; (3) measurements required to derive residence time estimates and experimental observations required for numerical simulations; and (4), with respect to models, (4a) the various numerical models and techniques employed to quantify and evaluate residence and flushing times and (4b) comparisons of Lagrangian versus Eulerian and internal versus external (post-processed) numerical particle tracking methods.

Approximately 60 people from academic, private industry and government organizations were identified and invited, based on their expertise, backgrounds and interests in the residence time topic. The actual workshop attendance was 34 people (see Attendees list, Appendix C). The workshop was structured around the three primary themes of *Applications*, *Measurements* and *Algorithms* (see Agenda, Appendix A) with speakers

invited to give brief talks in each of these theme areas (see Summaries of the Presentations, Appendix B). Following the first day of talks and follow-up discussions, breakout sessions were organized on each of the three theme areas with each session tasked with examining the theme, identifying shortcomings and recommending future focus to improve residence time estimation. The meeting concluded with the respective rapporteurs summarizing the outcomes of each of the respective sessions.

## 4. Workshop Results and Outcomes

The workshop opened with an introductory session (see Agenda, Appendix A) focused on the state of the art in residence time estimation, from the perspective of modeled transport and the concept of age (Cercio and Kim); to the variance in the definition of residence time found over a spectrum of estuaries (Jay); to the example of the Columbia River integrated observation and modeling system and its usefulness in characterizing residence time and other residual water properties (Baptista); and the challenge presented for Zinc in San Diego Bay (O'Connor). There were also discussions about whether calculations of residence times using Eulerian and Lagrangian methods are comparable. Often this may not be the case, because the processes to which these two approaches are applied are often different: Eulerian calculations for dissolved constituents usually treat the water-land boundary (including the bottom) as reflective, while for particulate materials this boundary is often treated as “sticky” or adsorptive (i.e. a sink). This session was followed by the three theme sessions focusing on applications, measurements and algorithms.

The applications presentations included an example of a completed observational and modeling effort at understanding the fate of copper in San Diego Bay (Chadwick); a summary of the challenges and needs for refined residence time estimates with respect to waste site response and remediation (Klein and White); and a succinct summary of the limits and challenges associated with particle tracking and dye release in calibrated models (Blumberg).

The session on measurements included a description of in situ tracer studies using the sulfur hexafluoride (SF<sub>6</sub>) tracer (Ho); a summary of dispersion estimates based on dye releases (Chant); and, as a lead in to the session on algorithms, a description of the impact of flooding and drying processes on numerical model-based estimates of residence time (Chen).

The algorithms session included a description of SqueezeBox, a flow-scaled 1-dimensional box model (Sheldon); the analysis of transport from synoptic Lagrangian maps (Wiggins); and the idea of generalized Lagrangian mean circulation (Hamrick).

A summary of results of the three themed breakout sessions follows.

#### **4a. Applications Session Summary**

For clarity, it became important to specify that this workshop is centered on flow-induced, hydraulic residence time. The group identified situations where knowledge of hydraulic residence time contributes to determining how long a chemical or particle would remain within a specified area. In almost no case was hydraulic residence time by itself considered sufficient information. Characteristics of the chemical or of the particle and its interaction with other substances can cause its residence time to be very different from that of a parcel of water. Nonetheless, estimating hydraulic residence time is an important part of an overall assessment of the fate and effects of man-made and natural inputs to any given space.

Applications of residence time include determining the fate of hazardous waste spills, rates of accumulation of chemical contamination from chronic point and non-point sources in sediment, steady-state aqueous concentrations of nutrients, calculations of total maximum daily loads of municipal waste, and rates of export of fish eggs and larvae released within a bay.

All estimates of residence times for such applications have been based, in effect, on box models with the precision of the estimate increasing as boxes become smaller and time scales shorter. The simplest estimates are sometimes all that is needed. For example, the relative susceptibilities of different estuaries to nutrient additions can be based on assuming whole estuaries to be single homogeneous boxes of a known volume. Residence time can then be defined by the annual average rate of freshwater addition.

When the concern centers on chemical contamination, the need is for residence time within small boxes rather than entire estuaries. When the fate of spills or the movement of fish eggs is at issue not only are small spatial scales important but also the time scale of interest is much less than a year. These finer scale estimates of residence time are included in the fine mesh numerical models. The grid cells associated with such models are small in the horizontal (of the order of tens of meters), may include vertical stratification, and the times can be nowcasts, based on real-time water level and wind data. Fine mesh hydrodynamic models are available for about twenty of the major estuaries in the United States. So long as an experienced practitioner is available to do so, the models readily provide precise estimates of rates of transport between cells (the inverse of residence time within cells). This dependence on experienced modelers is exacerbated by the fact that the existing models are from a variety of Federal and state agencies and academic groups, and each group uses different modeling techniques and computer codes. At the general level, a modeler can understand all the hydrodynamic models, but actually using any particular model requires experience with that model.

In the absence of an existing hydrodynamic model or of a modeler to use it and with a need for more spatial resolution than assuming estuarine-wide homogeneity, residence times can be estimated on the basis of the volume of the sub-estuarine areas and tidal prisms, fresh water inflow (if any), and salinity distributions (if not homogeneous).

## **4b. Measurements Session Summary**

Measurements (of, for example, salinity, currents, water levels) can be used for determining residence time and dispersion associated with estuaries provided that they include long-term data (for example, seasonal or at least a neap-spring tidal cycle). Measurements over only short times will lead to estimates that will miss important peaks in estuarine flushing cycles. In general, the minimum requirement for measurements is a longitudinal and cross-sectional salinity data set, which enables determination of the dispersion rate of an estuary; the diffusion coefficients can be determined from a tracer experiment. Even with the use of long-term data, the calculation of residence and flushing times is not straightforward. Extreme events such as storms resulting in strong winds and large river discharges may be the primary drivers of water transport in some areas but they can be entirely missed in even a long-term measurement program.

The major difficulties in conducting field measurements are the resources of instrumentation and skilled manpower to maintain it. Furthermore, for extreme events the use of autonomous instruments (e.g. autonomous underwater vehicles (AUV)) is more or less a necessity.

Before one can use numerical ocean models for the calculation of residence times, it is necessary to evaluate the models against measurements for any particular model application. With respect to model evaluation, it is necessary to define what is needed from the model (that is, the time and space scales of interest and resolution required) and to gain an understanding of the estuarine system before deciding what measurements are needed. The requirements are highly dependent on the location and the time scales associated with the modeling purpose. For many ecosystem management purposes, a set of 1-dimensional or 2-dimensional salinity and volumetric measurements over a multi-year time scale with a box model are sufficient to estimate residence time.

For both residence and flushing time computations and model validation, a suitable subset of required instrumentation would include: low cost salinity sensors, bio- and geo-chemical sensors for nutrients, high frequency radar for surface currents, AUVs, and acoustic Doppler current profilers for velocity and sediment transport measurements.

## **4c. Algorithms Session Summary**

Residence times can be quantified in terms of volume fluxes or particle fluxes (Burwell et al., 2000). Those from particle tracking are rare and there were questions concerning availability of such data in the literature. In terms of observations, there are dissolved tracer concentration data but much less in the way of particle/drifter data.

Lagrangian particle tracking is attractive because the particles have information associated with them (velocities, positions, temperature, etc.). However, a particle field and a tracer/concentration field would evolve differently in time because the latter inherently averages over a lower level of spatial dispersion. Realistic particle tracking

simulations should include some measure of dispersion (which is not trivial to determine, see Blumberg et al., 2004)) because experiments show that two floats/drifters placed at the same initial spatial location will follow different trajectories due to these dispersive effects. The tracking of particles can be performed either inside a numerical algorithm (internally) or externally as a post-processing exercise; some numerical studies have shown that the use of hourly numerical solutions enables the post-processing approach to yield results indistinguishable from those of the internal approach. It was also decided that for the post-processing approach, it is hourly instantaneous particle data that should be used and not hourly-averaged data. There was also an interest expressed in the back-tracking (back in time) of particles and related algorithms (but they were not discussed) and the post-processing approach is ideally suited for this exercise.

It was decided that the comparison of a simple laboratory model with predictions from various numerical ocean models was not worthwhile because it is already known that models can reproduce simplified and idealized laboratory setups relatively accurately. Residence time calculations for the realistic and more complex applications are numerically challenging and are of greater interest. For such applications, the spatial domains of interest (for example, size of a bay, estuary or segment there of) are of critical importance and the residence times are expected to vary widely depending on the choices made. In cases where only average residence times over large spaces are needed, a box model calculation is sufficient and sophisticated numerical ocean models are not needed.

It was proposed that perhaps the most reasonable ways to achieve residence time estimates through the use of numerical ocean circulation models are via:

1. The analysis of residual velocity and salinity intrusion fields over long durations, and
2. The dynamical systems approach (Wiggins et al., 2005) where convergent/divergent zones are visible on the Synoptic Lagrangian Maps, although this approach requires the (pre) determination of some parameter values.

Both these approaches require large numerical data sets (for example, three-dimensional velocity and tracer output fields from an ocean model also spanning a wide interval in time) containing much fine-resolution (in space and time) information. In this respect, these approaches may be regarded as being numerically the most rigorous and precise ways to estimate residence times.

## **5. Summary, Recommendations and Future Directions**

Residence times can be estimated using the following approaches:

- (a) direct measurements (as in section 4b);
- (b) box models (and their relatively more sophisticated extensions – see Introduction);
- (c) hydrodynamic models (using concentration patches and/or particles);

- (d) residual velocity and salinity intrusion methods (using observed data and/or hydrodynamic model outputs); and
- (e) the dynamical systems approach (via Synoptic Lagrangian Maps using hydrodynamic model outputs).

If the concern is for the average condition in an entire bay or estuary, then simplified models (for example, (a) and (b) above) are adequate but they will greatly underestimate or fail to address the residence times associated with smaller regions, such as urban harbors and embayments, which can exhibit restricted circulation. The more sophisticated numerical ocean models can be the basis of improved estimates of residence time in these small sub-embayments. With respect to (a), (b) and (d) above, cross-sectional salinity and long-term data on fresh water input are desirable, although this is not always feasible and in such cases numerical model output may be used as a proxy. Numerical models themselves can have problems during extreme events (e.g. during strong storm wind and river discharge events) and in such situations (a) and (b) are the most realistic options.

Residence time research and analysis now involves the application of physically more sophisticated 3-dimensional circulation models (Burwell et al., 2000; Shen and Haas, 2004) to sub-domains of bays and estuaries of arbitrary size (resulting in 3-dimensional gridded flow-field data over multiple time levels), and high resolution estimates of residence times can be achieved via (i) the concentration patch approach, (ii) the particle tracking approach (done internally or externally of an ocean model), (iii) the residual velocity and salinity intrusion approach and (iv) the Synoptic Lagrangian Map approach. The particle tracking approach requires the inclusion of some level of artificial dispersion in keeping with the levels of horizontal diffusion present in the ocean; hence, it may be necessary to examine several different dispersion-diffusion (Brownian motion related) formulations before selecting the most appropriate choice. The particle-related dispersion should be such that the particle paths are consistent with the dynamics of a passive tracer patch (Blumberg et al., 2004) and therefore, some cross-calibration tests between (i) and (ii) above may also need to be performed (as in Burwell et al., 2000) in order to verify this relationship. Using the above approaches, it is possible to construct maps of the variations of residence times in time and space over whole bays and estuaries; inhomogeneities in the distributions of these residence times will provide vital evidence in support of important locally occurring phenomena.

It would be a worthwhile exercise to compare, contrast and document the estimates of residence times resulting from the above described different approaches for some well known bays and estuaries. Provided that there exist complete and quality controlled data sets for these bays and estuaries, residence time estimates can be calculated (1) directly from them (using for example, the simpler box models) and (2) from hydrodynamic model outputs where the models have been driven using this data. Such data sets will also serve to enable the determination of the artificial dispersion coefficients required in the tracking of Lagrangian particles. Special studies of interest could be (a) tidal (multiple entrant) and non-tidal situations and (b) situations involving variable surface forcings – from calm weather conditions to extreme conditions such as hurricanes. Due to the diverse and extensive nature of such a major exercise, it could perhaps form the

basis for a multi-institutional collaborative effort with each institution specializing in a particular facet of the project.

## **Acknowledgements**

The authors would like to thank all the participants in the workshop for a stimulating and provocative discussion on the topic of transport time scales in bays and estuaries. We are especially grateful to the speakers and, in particular, we would like to thank Joan Sheldon and Ed Dettmann for their significant critique and input to this report.

## References

- Andrews J. C and H. Muller. 1983. Space-time variability of nutrients in a lagoonal patch reef. *Limnology and Oceanography*, 28, 215-227.
- Basu B. K. and F. R. Pick. 1996. Factors regulating phytoplankton and zooplankton biomass in temperate rivers. *Limnology and Oceanography*, 41, 1572-1577.
- Blumberg A. F., Dunning D. J., Li H., Heimbuch D. and W. R. Geyer. 2004. Use of a particle-tracking model for Predicting Entrainment at Power Plants on the Hudson River, *Estuaries*, 27(3), 515-526.
- Boynton W. R., Garber J. H., Summers R. and W. M. Kemp. 1995. Inputs, transformations, and transport of nitrogen and phosphorus in Chesapeake Bay and selected tributaries. *Estuaries*, 18, 285-314.
- Bricelj V. M. and D. J. Lonsdale. 1997. *Aureococcus anophagefferens*: Causes and ecological consequences of brown tides in U. S. Mid-Atlantic coastal waters. *Limnology and Oceanography*, 42, 1023-1038.
- Burwell D., Vincent M., Luther M. and M. Galperin. 2000. Modeling Residence Times : Eulerian vs. Lagrangian. In : *Estuarine and Coastal Modeling*, M. L. Spaulding and H. L. Butler, eds., ASCE, Reston, VA, 995-1009.
- Dettmann E. H. 2001. Effect of water residence time on annual export and denitrification of nitrogen in estuaries : A model analysis, *Estuaries*, 24 (4), 481-490.
- Hagy, J. D., L. P. Sanford, and W. R. Boynton. 2000. Estimation of net physical transport and hydraulic residence times for a coastal plain estuary using box models. *Estuaries*, 23(3), 328-340.
- Miller, R. L. and B. F. McPherson. 1991. Estimating estuarine flushing and residence times in Charlotte Harbor, Florida, via salt balance and a box model. *Limnology and Oceanography*, 36, 602-612.
- Monsen N. E., Cloern J. E. and L. V. Lucas. 2002. A comment on the use of flushing time, residence time and age as transport time scales. *Limnology and Oceanography*, 47(5), 1545-1553.
- Painchaud J., Lefavre D., Therriault J. -C. and L. Legendre. 1996. Bacterial dynamics in the upper St. Lawrence estuary. *Limnology and Oceanography*. 41, 1610-1618.
- Schmalz, R. 2004. Development of the NOS Galveston Bay Operational Forecast System. *Proceedings, 7<sup>th</sup> International Marine Environmental Modelling Seminar*, Washington, DC.

Seitzinger, S. 2000. Scaling up : Site-specific measurements to global estimates of denitrification, *Estuarine Science : A Synthetic Approach to Research Practice*, John E. Hobbie (Ed), Island Press, Washington DC.

Sheldon, J. E., and M. Alber. 2002. A comparison of residence time calculations using simple compartment models of the Altamaha River estuary, Georgia. *Estuaries* **25**: 1304-1317.

Shen J. and L. Haas. 2004. Calculating age and residence time in the tidal York River using three-dimensional model experiments, *Estuarine Coastal and Shelf Science*, **61**, 449-461.

Signell, R.P and B. Butman. 1992. Modeling tidal exchange and dispersion in Boston Harbor. *J. Geophys. Res.*, 97, 15191-15606.

Takeoka H. 1984. Fundamental concepts of exchange and transport time scales in a coastal sea, *Continental Shelf Research*, **3**, 311-326.

Wei, E. 2003. The new Port of New York and New Jersey Operational Forecast System. *Bulletin of the American Meteorological Society*, 84 (9), 1184-1186.

Wei, E., T. Caplow and P. Schlosser. 2004. Solute dispersion modeling in New York Harbor. *NOAA Technical Report NOS CS 18*, 45p.

Wiggins, S. R. and D. Small. Synoptic Lagrangian Maps and Lagrangian particle tracking, *J. Dynamical Systems*, **100(5)**, 1000-1010.

Zimmerman, J. T. F. 1976. Mixing and flushing of tidal embayments in the western Dutch Wadden Sea. Part I: Distribution of salinity and calculation of mixing time scales. *Neth. J. Sea Res.* **10**: 149-191.

# Appendix A : Workshop Agenda

## NOS Workshop on Residence/Flushing Times in Bays and Estuaries

June 8-9, 2004  
1315 East-West Highway, SSMC3, 4<sup>th</sup> Floor Conference Room  
NOAA, Silver Spring, MD 20910

### **Tuesday, June 8**

#### Overview (9-10:30am)

Frank Aikman - Introductions and charge to workshop

#### Current State of the Art

Carl Cerco and Sung-Chan Kim – RT estimates from modeled transport and the concept of “age”

David Jay – Some thoughts on the concept of residence time

Antonio Baptista – The CORIE integrated observation and modeling system and its contribution to characterizing residual properties and RT

Tom O’Connor – Modeling zinc in San Diego Bay

#### Applications (11-12:30pm)

Bart Chadwick – Modeling the fate of copper in San Diego Bay

Lawrence Klein and Jessica White – Waste site response and remediation

Alan Blumberg – Determining estuarine residence/flushing times

#### Measurements (1:30-3:00pm)

David Ho - SF<sub>6</sub> tracer release experiments

Bob Chant – Lagrangian observations of mixing in a stratified estuary

Changsheng Chen – The impact of the flooding/drying process on the residual time in estuaries

#### Algorithms (3:30-5:00pm)

Joan Sheldon – SqueezeBox: Flow-scaled 1-D box models for estuary residence time estimates

Steve Wiggins – Transport analysis using Synoptic Lagrangian Maps (SLMs): the dynamical systems approach

John Hamrick – Generalized Lagrangian Mean (GLM) theory revisited

### **Wednesday, June 9**

#### Breakouts (8:00-10:00am)

Three teams, charged with examining each theme and making recommendations back to workshop (using: examples; problems; possible solutions)

Rapporteurs:

Applications – Tom O’Connor

Measurements – Eugene Wei  
Algorithms – Lyon Lanerolle

Breakout Team Reports/Discussion (10:30-12:00pm)  
Adjourn (12:30pm)

## **Appendix B : Summaries of the Presentations**

### **Introduction and charge to the workshop (Frank Aikman III)**

Beginning with a definition of Residence Time (RT), the following two questions were posed: *(i)* is residence time more appropriate to describe times associated with particles or particle ensembles? and *(ii)* is flushing time best suited for times associated with tracer concentrations? Thereafter, the objectives of the workshop were stated as the dissemination of the current state of the art in residence times (techniques, methods, etc.), applications of residence times, measurements/observations and their use in estimating residence times and algorithms for calculating residence times (both Eulerian and Lagrangian approaches). Finally, the desired outcomes were stated as numerical model intercomparisons, “who needs what?” (for example, chemists, biologists, etc.), a workshop report (with potential reviewers and editors) and “what is next?” (current shortcomings and future recommendations).

### **RT estimates from modeled transport and the concept of “age” (Carl Cerco & Sung-Chan Kim)**

Several definitions for residence and flushing times were provided via the first moment of concentration, salinity concentration variation (with observed data), age (using a partial differential equation), dye releases and particle path tracking approaches. These various estimates were then compared and contrasted for Chesapeake Bay. In conclusion, it was said that *(i)* no single time scale describes all of the transport processes and *(ii)* no single transport time scale is valid for all time periods and spatial locations.

### **Some thoughts on the concept of residence time (David Jay)**

Residence times are not a single number for a system but they do vary both in space and time and are different for different substances. They are a function of the type of system, domain bathymetry, tides and various events (hurricanes, floods, etc.). Fjords for example, have highly variable residence times in space and time. Some demonstrations of the variations of residence times using suspended particulate matter in the Columbia river were discussed.

### **The CORIE integrated observation and modeling system and its contribution to characterizing residual properties and RT (Antonio Baptista, Michela Burla and Anabela Oliveira)**

First the CORIE observation network was described and thereafter residence time was defined in an Eulerian (flushing based) and Lagrangian (particle path tracking) framework with the latter covering both a “once through” and a re-entrant scenario. The question “will coastal observations change how we evaluate RT?” was raised and to which the answer turned out to be “yes, much yet to be determined”. Finally, the accuracy of observations was discussed together with the ensuing residual velocities and salinity intrusions.

### **Modeling zinc in San Diego Bay (Tom O'Connor)**

Given a known input rate of zinc into San Diego Bay, the zinc concentration in the bay was calculated by first assuming complete and total mixing so that zinc was at a uniform concentration throughout the bay, regardless of locations of sources. Zinc left the bay with the tidal prism and steady state zinc concentration was simply that required for the output rate to equal the input rate. Given that the calculated steady state zinc concentration was well below water quality criteria the simplifying assumptions may have been justified.

### **Modeling the fate of copper in San Diego Bay (D. Bart Chadwick, Igancio Rivera-Duarte, Alberto Zirino, Amy Blake and Chuck Katz)**

The total copper loading for San Diego bay was described and the balance and fate of copper was modeled using a tidally-averaged, 1D box model (non-conservative, steady-state) and a predictive 2D model (non-conservative, non-steady-state). The best results for the 1D model were obtained by assuming a uniform settling rate loss configuration. The calibrated (against data) 2D model improved the spatial and temporal resolution and provided a useful management tool for evaluating changes to copper loading in San Diego bay.

### **Waste site response and remediation (Lawrence Klein and Jessica White)**

An introduction to the role and responsibilities of the NOS Coastal Protection and Restoration Division was given and some of its needs were addressed (better techniques for source identification, better prediction of injury and understanding effects of dredging, etc.). Thereafter, assessment of injury due to contamination, waste site challenges (and how to address them and the information needed to do so) and model utilization (and model capabilities and inputs needed – currents, rivers, tides, winds, etc.) were discussed.

### **Determining estuarine residence/flushing times (Alan Blumberg and Quamrul Ahsan)**

Flushing or residence time of an estuarine system can be quantified using two physically consistent approaches. They are transport modeling of conservative substances and particle tracking. Mathematically these two methodologies are identical. The transport modeling approach solves a mass balance equation for a conservative substance, whereas, the particle tracking methodology is a Lagrangian approach that tracks the movement of individual particles with time. The latter approach has some inherent constraints that lead to an adoption of complex treatments of boundary conditions. Other constraints include the interpolation methods of velocity at the particle position, the number of particles used in the analysis, the role of dispersion, the uncertainty in the analysis and the validation process. The complexity in particle tracking modeling has been significantly resolved by configuring the methodology in the ECOM (Estuarine and Coastal Ocean Model, a derivative of the Princeton Ocean Model) framework. The ECOM model has been the model of choice for almost 3000 scientists and engineers during the past 25 years. The

particle tracking methodology has been adopted by us in modeling the New York/New Jersey Harbor Estuary with considerable success. There are primary challenges still to be addressed of which the validation of particle path tracking (for realistic simulations) and obtaining stable statistics (with a computationally reasonable ensemble particle population) are some examples.

#### **SF<sub>6</sub> tracer release experiments (David Ho, Ted Caplow and Peter Schlosser)**

Sulfur hexafluoride (SF<sub>6</sub>) is an ideal substance to use in tracer experiments because it is inexpensive, inert and non-toxic. Some tracer experiments in the Hudson River, Newark Bay and East River/Long Island Sound areas were described together with details on the injection and measurement techniques. The resulting measurements were compared with numerical predictions for the years 2001 and 2003 and residence time estimates were also derived and compared between the observed data and the numerical predictions.

#### **Lagrangian observations of mixing in a stratified estuary (Bob Chant, Rocky Geyer, Bob Houghton, Eli Hunter and Jim Lerczak)**

An expression for effective dispersion was derived using a salinity gradient and a salinity flux (or a mean velocity times a mean salinity) and it was used to obtain an expression for the residence time in an estuary using flux exchange arguments. This expression was thereafter applied to a simple channel in order to obtain a residence time value for it in terms of the effective dispersion. Experiments in the Hudson River in May 2002 showed that the tides were the mixing agent and also that the tidally mean flow did remain constant over the neap/spring cycle. Dye experiments yielded the velocity patch structure. Together with salinity measurements, it was possible to calculate an effective dispersion thereby leading to potential estimations of residence times.

#### **The impact of the flooding/drying process on the residual time in estuaries (Changsheng Chen and Haosheng Huang)**

Several definitions for residence and flushing times were presented with the relatively newer dynamical tracer method favored for application in this research. The surface residual flow and residual mass were computed after applying the FVCOM model to a domain near Broad River, SC where wetting/drying was important. Using the residual masses, residence times were computed for two scenarios (with and without inter-tidal salt marshes) which leads to the conclusion that the inclusion of the flooding/drying process in an estuarine model tended to shorten residence times and hence enhanced the water exchange.

#### **SqueezeBox : Flow-scaled 1D box models for estuary residence time estimates (Joan Sheldon and Merryl Alber)**

SqueezeBox is a desktop modeling tool for estimating longitudinal salinity distributions and mixing time scales, exploring transport of inert tracers, and evaluating the effects of freshwater inflow in riverine estuaries. It generates 1-dimensional, tidally averaged box models with structures scalable for different river flows. Scaling the models for numerical stability, a step not always included for simple box models, allows them to be

used for simulations of transient conditions such as those following a pulse input of a non-reactive substance (tracer) dissolved in the water. These features allow for estimation of a variety of mixing time scales (e.g. transit time and residence times) and for investigation of the duration of exposure of estuary segments to introduced substances. These types of predictions from SqueezeBox may be useful in interpreting nutrient and pollutant dynamics in estuaries or in evaluating the exceedance of water quality standards. It could also be used to compare the relative susceptibility of different estuaries to perturbations. The modeling framework is flexible, so that new modules can be developed independently of the application, and modules have minimal data requirements. We have completed modules for two riverine estuaries in Georgia: the Ogeechee, a slower-flowing estuary with primarily coastal plain drainage, and the Altamaha, a faster-flowing estuary with extensive piedmont drainage.

**Transport analysis using Synoptic Lagrangian Maps (SLMs) : the dynamical systems approach (Steve Wiggins, Des Small, Denny Kirwan, Bruce Lipphardt, C. E. Grosh and J. Paduan)**

Synoptic Lagrangian Maps (SLMs) encapsulate the spatio-temporal structure of the future and/or history of particle trajectories. They are useful when large velocity field archives are available, for example velocity vectors on a grid for multiple increments in time which could come from either observed data or numerical model predictions. In these maps, each particle trajectory is represented by a single pixel, color-coded according to its residence time and/or information about its origin/fate. There are many kinds of SLMs : forward/backward in time, those containing multiple boundary segments, those containing spatio-temporal information, those with forward/backward/total residence times, those giving fate location, those giving transport pathways, etc. Dynamical systems theory (for example, Lyapunov exponents) helps to describe the structures that organize qualitatively different particle trajectories and provides the building blocks of the geometric template for Lagrangian transport (for example, lobe dynamics which can be used to investigate flow barriers, transport alleyways, water mass exchange, etc.).

**Generalized Lagrangian Mean (GLM) theory revisited (John Hamrick)**

Generalized Lagrangian Mean (GLM) theory is an exact theory of wave-mean flow interaction and it is an Eulerian mean along a Lagrangian trajectory which is assigned to a mean position associated with the trajectory. In GLM theory, quantities are mathematically manipulated using Eulerian mean (averaging) operators and the practical implementation is usually highly non-linear. For Lagrangian particle path tracking, the following sequence of events is adopted : (i) release particles, distributed uniformly in space, at numerous intervals over a pre-specified averaging period, (ii) track each particle for a period equal to the averaging period, (iii) determine in place the associated mean position and velocity of the particle along the trajectory, (iv) average the trajectory mean positions and velocities of each particle having the same initial position, (v) locate the average velocities at the average position, (vi) interpolate these velocities back to the hydrodynamic model grid points, (vii) assume that the form of the weakly non-linear GLM continuity is applicable to more non-linear situations, (viii) equate the particle tracked GLM plus an error in the form of a scaled potential to the GLM transport, (ix)

clean the divergence from the particle tracking GLM velocity by solving for the scalar potential and  $(x)$  finally, a GLM transport field results which satisfies an Eulerian form of a continuity equation. Therefore, GLM theory has the potential for the Eulerian analysis and display of Lagrangian processes.

## Appendix C : Attendees

Frank Aikman III  
CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2809x101  
[Frank.Aikman@noaa.gov](mailto:Frank.Aikman@noaa.gov)

Antonio Baptista  
OGI School of Science and Engineering  
Oregon Health and Science University  
20000 NW Walker Road  
Beaverton, OR 97006  
(503) 748-1147  
[baptista@cclmr.ogi.edu](mailto:baptista@cclmr.ogi.edu)

Alan F. Blumberg  
Department of Civil, Environmental & Ocean Engineering  
Stevens Institute of Technology  
Castle Point on Hudson,  
Hoboken, NJ 07030  
(201) 216-5289  
[ablumber@stevens.edu](mailto:ablumber@stevens.edu)

D. Bart Chadwick  
SPAWAR Systems Center San Diego  
Code 2362  
53475 Strothe Road  
San Diego, CA 95152  
(619) 553-5333  
[chadwick@spawar.navy.mil](mailto:chadwick@spawar.navy.mil)  
[bart.chadwick@navy.mil](mailto:bart.chadwick@navy.mil)

Carl Cerco  
US Army Research and Development Center  
Waterways Experiment Station  
3909 Halls Ferry Road  
Vicksburg, MS 39180-6199  
(301) 634-4207  
[cercoc@wes.army.mil](mailto:cercoc@wes.army.mil)  
[Carl.F.Cerco@erdc.usace.army.mil](mailto:Carl.F.Cerco@erdc.usace.army.mil)

Bob Chant  
Institute of Marine and Coastal Sciences  
Rutgers University

71 Dudley Road  
New Brunswick, NJ 08901  
(732) 932-6555 x 544  
[chant@marine.rutgers.edu](mailto:chant@marine.rutgers.edu)

Changsheng Chen  
School of Marine Science and Technology  
University of Massachusetts - Dartmouth  
706 South Rodney French Boulevard  
New Bedford, MA 02744  
(508) 910-6388  
[c1chen@umassd.edu](mailto:c1chen@umassd.edu)

Edward Dettmann  
EPA/NHEERL  
27 Tarzwell Drive  
Narragansett, RI 02882-1154  
(401) 782-3039  
[dettmann.edward@epamail.epa.gov](mailto:dettmann.edward@epamail.epa.gov)

Jerome Fiechter  
RSMAS – Applied Marine Physics  
4600 Rickenbacker Causeway  
Miami, FL 33149  
(305) 361-4984  
[jfiechter@rsmas.miami.edu](mailto:jfiechter@rsmas.miami.edu)

Tom Gross  
CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2809x139  
[Tom.Gross@noaa.gov](mailto:Tom.Gross@noaa.gov)

John M. Hamrick  
Tetra Tech, Inc.  
10306 Eaton Place, Suite 340,  
Fairfax, VA 22030  
(703) 385-6000  
[John.Hamrick@tetrattech-ffx.com](mailto:John.Hamrick@tetrattech-ffx.com)

Ferdi Hellweger  
842a Mudd  
Earth & Environmental Engineering  
Columbia University  
New York, NY 10027  
(917) 238-3010  
[flh23@columbia.edu](mailto:flh23@columbia.edu)

David Ho  
Lamont-Doherty Earth Observatory of Columbia University  
Geochemistry Division  
69 Geochemistry  
61 Route 9W – PO Box 1000  
Palisades, NY 10964  
(845) 365-8706  
[david@ldeo.columbia.edu](mailto:david@ldeo.columbia.edu)

David Jay  
OGI School of Science and Engineering  
Oregon Health and Science University  
20000 NW Walker Road  
Beaverton, OR 97006  
(503) 748-1372  
[djay@ebs.ogi.edu](mailto:djay@ebs.ogi.edu)

Sung-Chan Kim  
US Army Corps of Engineers  
Engineering Research and Development Center  
Environmental Laboratory  
CEERD-EP-W  
3909 Halls Ferry Road  
Vicksburg, MS 39180  
(601) 634-3783  
[Sung-Chan.Kim@erdc.usace.army.mil](mailto:Sung-Chan.Kim@erdc.usace.army.mil)

Dennis Kirwan  
103 Robinson Hall  
University of Delaware  
Newark DE 19716  
[adk@udel.edu](mailto:adk@udel.edu)  
[adk@smash.cms.udel.edu](mailto:adk@smash.cms.udel.edu)

Lawrence Klein  
Associate Coastal Resource Coordinator  
Coastal Protection and Restoration Division  
NOAA/NOS/Office of Response and Restoration  
c/o USEPA (6SF-L), 1445 Ross Avenue  
Dallas, TX 75202-2733  
[Lawrence.Klein@noaa.gov](mailto:Lawrence.Klein@noaa.gov)

Lyon Lanerolle  
CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2809x110

[Lyon.Lanerolle@noaa.gov](mailto:Lyon.Lanerolle@noaa.gov)

Bruce Lipphardt  
104 Robinson Hall  
University of Delaware  
Newark DE 19716  
[brucel@udel.edu](mailto:brucel@udel.edu)  
[brucel@smash.cms.udel.edu](mailto:brucel@smash.cms.udel.edu)

Daniel G. MacDonald,  
School of Marine Science and Technology  
University of Massachusetts - Dartmouth  
706 South Rodney French Boulevard  
New Bedford, MA 02744  
(508) 910-6334  
[dmacdonald@umassd.edu](mailto:dmacdonald@umassd.edu)

Reza Malek-Madani  
Research Office, MS 10m  
589 McNair Road  
U. S. Naval Academy  
Annapolis MD 21402  
[research@usna.edu](mailto:research@usna.edu)  
[malekmr@onr.navy.mil](mailto:malekmr@onr.navy.mil)

Buzz Martin  
The Texas General Land Office  
1700 N. Congress Avenue, Suite 840  
Austin TX 78701  
[Buzz.Martin@GLO.STATE.TX.US](mailto:Buzz.Martin@GLO.STATE.TX.US)

Tom O'Connor  
NCCOS/NOS/NOAA  
1305 East-West Highway  
Silver Spring, MD 20910  
(301) 713-3028x151  
[Tom.Oconnor@noaa.gov](mailto:Tom.Oconnor@noaa.gov)

Bruce Parker  
CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2801x121  
[Bruce.Parker@noaa.gov](mailto:Bruce.Parker@noaa.gov)

Debbie Payton  
NOAA/NOS/Office of Response and Restoration  
7600 Sand Point Way NE

Seattle, WA 98115  
(206) 526-6320  
[Debbie.Payton@noaa.gov](mailto:Debbie.Payton@noaa.gov)

Hayder Salman  
Mathematics Department  
CB #3250, Phillips Hall  
UNC – Chapel Hill  
Chapel Hill, NC 27599  
(919) 962-1294  
[hsalman@email.unc.edu](mailto:hsalman@email.unc.edu)

Richard Schmalz  
CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2809x104  
[Richard.Schmalz@noaa.gov](mailto:Richard.Schmalz@noaa.gov)

Joan Sheldon  
Department of Marine Sciences  
University of Georgia  
Athens, GA 30602  
(706) 542-1283  
[jsheldon@uga.edu](mailto:jsheldon@uga.edu)

Craig Swanson  
Applied Science Associates, Inc.  
70 Dean Knauss Drive  
Narragansett, RI 02882  
(401) 789-6224  
[cswanson@appsci.com](mailto:cswanson@appsci.com)

Arnoldo Valle-Levinson  
Center for Coastal Physical Oceanography  
Old Dominion University  
Norfolk, VA 23529  
(757) 683-5578  
[arnoldo@ccpo.odu.edu](mailto:arnoldo@ccpo.odu.edu)

Glen Watabayashi  
NOAA/NOS/Office of Response and Restoration  
7600 Sand Point Way NE  
Seattle, WA 98115  
(206) 526-6324  
[Glen.Watabayashi@noaa.gov](mailto:Glen.Watabayashi@noaa.gov)

Eugene Wei

CSDL/NOS/NOAA  
1315 East-West Highway  
Silver Spring, MD 20910  
(301) 713-2809x102  
[Eugene.Weil@noaa.gov](mailto:Eugene.Weil@noaa.gov)

Jessica White  
Associate Coastal Resource Coordinator  
Coastal Protection and Restoration Division  
NOAA/NOS/Office of Response and Restoration  
c/o US EPA (6SF-L), 1445 Ross Ave.  
Dallas, Texas 75202-2733  
[Jessica.White@noaa.gov](mailto:Jessica.White@noaa.gov)

S. R. Wiggins  
Department of Mathematics  
University of Bristol  
University walk  
Clifton  
Bristol BS8 1TW  
United Kingdom  
+44 117 928 7979  
[S.Wiggins@bristol.ac.uk](mailto:S.Wiggins@bristol.ac.uk)