

# A Derived Model of Alaskan Sea Surface Topography: a critical piece of the vertical datum transformation for ellipsoid referenced hydrography

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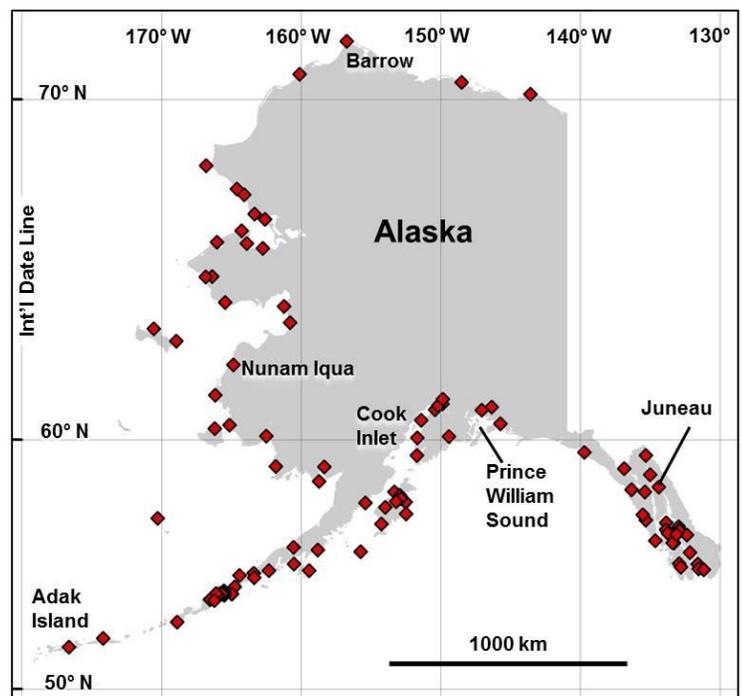
## Abstract

With coasts that form three >1700 km arms which spiral outward from a point near Kodiak Island, Alaska provides a large-scale site for the study of the relationship between geodesy and sea level. The surface of a static, uniform ocean will follow an equipotential, orthometric surface. Long lived geostrophic currents or prevailing winds result in localized and large scale variations between its surface and the geoid datum. The topography of the sea surface (TSS) relative to the geoid is a critical component for conducting hydrographic survey operations. We modeled TSS in Alaska by examining locations which had both historical tidal datums and a GPS occupation of a nearby benchmark. From observations at over 100 such stations throughout Alaska, we observed that the sea surface forms a SW dipping slope which is ~1.4 m above NAVD88 (GEOID12B) near the SE corner of Alaska, and ~35 cm above NAVD88 near Adak Island ~2700 km to the east. Point observations of TSS are extended to all regions via a Laplacian interpolation. This TSS model, in conjunction with the Geoid12B model and a model of tidal amplitudes can be summed to derive a datum separation between the GPS reference ellipse and chart datum.

**Keywords**— *Alaska hydrography; topography of the sea surface; tidal datum; VDatum;*

## Introduction

Using Global Navigation Satellite Systems (GNSS) for vertical positioning and control for hydrographic surveys has a number of significant advantages [1] and NOAA Office of Coast Survey intends to conduct all surveys to the GNSS control. The general problem of working with GNSS, or ellipsoid based positioning, can be broken into two parts: obtaining trajectories relative to the ellipsoid, and establishing the vertical datum transformations between the ellipse and the chart datum. This paper addresses a critical part of the later. NOAA's VDatum project uses a combination of water level observations, hydrodynamic modeling, and geoid models to calculate these transformations [1]. Regional VDatum models have been released for the continental United States [2], but because of a lack of data, have not yet been released for Alaska.



**Figure 1: Study area. Red diamonds represent locations of tidal datum stations used.**

In areas without established vertical datum transformations, like Alaska, some of the advantages of GNSS vertical control can still be realized through approaches that leverage some aspects of water-level control such as Ellipsoid Referenced Zoned Tides (ERZT) [1]. Another promising approach is Ellipsoid-Referenced Zoned Datum or Poor Man’s VDatum [3], which extends the ERZT idea to develop a model of the datum separation by using a geoid model, a model of the Topography of the Sea Surface (TSS), and traditional hydrographic water levels packages such as zoned tides or Tidal Constituent and Residual Interpolation (TCARI) [4] models. This paper outlines the development of this TSS model. Importantly, the TSS model presented here is developed specifically for this application, and should not be applied to other potential uses, such as understanding ocean currents, without understanding the limitations of our approach. That is, we make no claim that this is a physically reasonable model of TSS; it is a model that works with the implementation of datum estimation outlined in [3].

The average surface of a static, uniform ocean will follow an equipotential gravitational surface. However, long-lived ocean currents or prevailing winds can result in localized or large scale variations between the sea surface and an equipotential surface. This difference between the long-term average water level and the geoid is the TSS. We modeled TSS in Alaska by examining locations which had both historical tidal datums and a GNSS occupation of a nearby benchmark and then modeled values between stations through Laplacian interpolation. We measured MSL relative to the Geoid12B [5], a hybrid geoid that is equivalent to NAVD88 for this purpose.

## Methods

The State of Alaska has coastlines that cover a large area extending from near the International Date Line to 130° west, and from 51° to 71° north (fig. 1). Within the state, there are over 250 locations where tidal datums have historically been determined and are available through NOAA’s Center for Operational Oceanographic Products & Services (CO-OPS) [6]. Here, we chose data from 26 active tidal stations and 79 non-active tidal datum locations. We choose those sites that had benchmarks with recent GPS height observations available through NOAA’s Online Positioning User Service (OPUS) system [7]. While this analysis would be trivial if the National Geodetic Survey (NGS) database containing GPS observations were linked to those from the Center for Coastal and Ocean Products (CO-OPS) containing tidal datum information, they are not. The matching of benchmarks was achieved through stamping designators, descriptions, and locations. We suspect that additional benchmarks with both calculated tidal datums and ellipsoid heights may be recoverable from other archives. Outlying data from four stations were discarded due to the age of the station survey or because their water level data appear to be significantly influenced by their proximity to a major river.

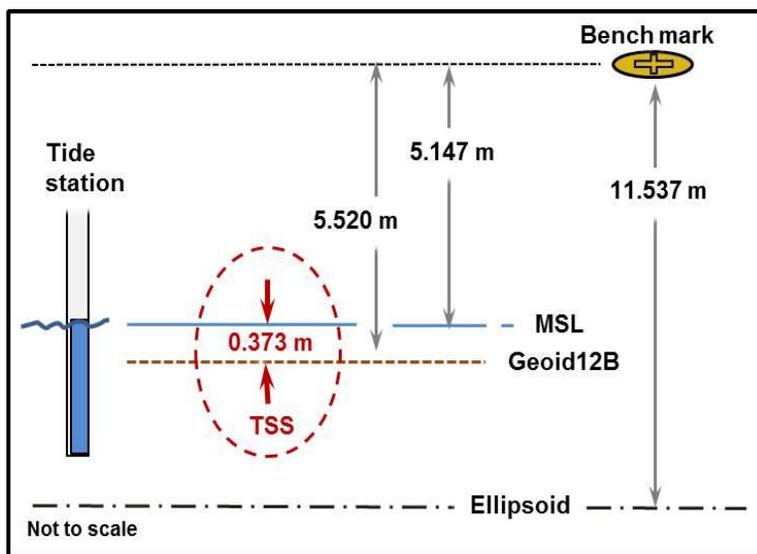


Figure 2: example of tide station reference data from Adak Island, AK. Benchmark UW7919 is measured relative to the NAD83 ellipsoid and NAVD88 Geoid12B by GNSS occupation. The benchmark is also leveled to the tide gauge datum then to mean sea level (MSL). The

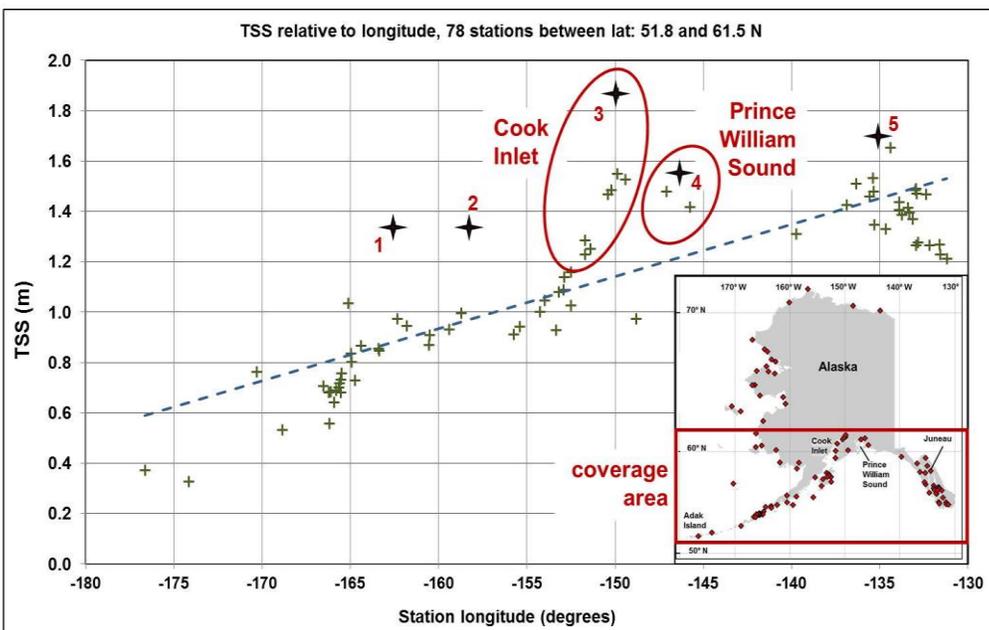
For each tidal datum location, the mean water levels are reported relative to the station’s vertical datum, and a benchmark report gives the relative heights of the benchmarks relative to station datum, typically measured with differential optical leveling. The GNSS derived height of one of these benchmarks is extracted from the submitted OPUS solutions. This ellipsoid height of the benchmark allows us to tie together the water level datums and the ellipsoid referenced Geoid 12B heights. The TSS is then simply the difference between the MSL and the modeled Geoid12B height relative to the ellipsoid at that location (Fig. 2). We recognize that any errors in the Geoid 12B model, which is known to have serious defects in Alaska, will be embedded in the SST calculated by this method.

The point determination of TSS were then extracted using Laplace interpolation. The Laplace equation minimizes the integrated square of a harmonic function’s square gradient and the solution is unique and always exists. The solution to the Laplace equation may thus be regarded as a “perfect interpolator”, producing values that are in-line with the available known information. The Laplace interpolator we use for TSS is as implemented in TCARI. TCARI encapsulates the Laplace interpolation engine into a solver framework that takes into account the influence of landforms (shoreline and islands) and also accommodates free “ocean bounds” At all locations in this study area, MSL is higher than the geoid surface.

The sensitivity of the modeled TSS to individual stations was analyzed using a cross validation or leave-one-out analysis. The interpolation model was run omitting each station and the modeled TSS at the station location using all remaining stations was compared to the measured TSS at that station.

## Results

The interpolated TSS exhibits a prominent, west-southwest dipping trend (figs. 3, 4). The TSS height is

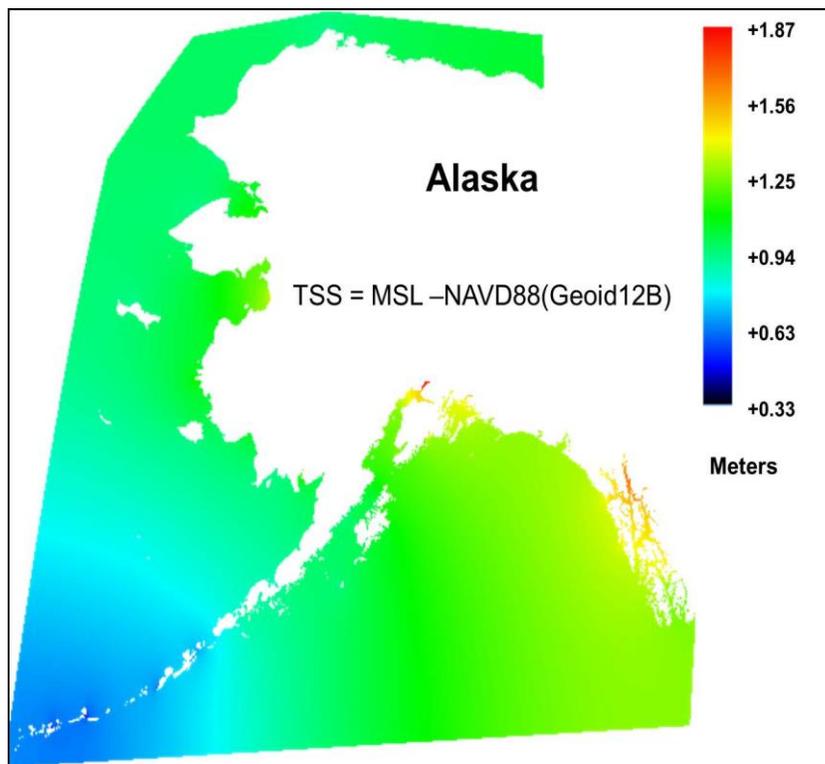


**Figure 3: TSS trend from east to west for Alaskan stations south of 61.5° N. Latitude. See inset map for location. Enhanced data points represent stations within a confined channel with mean water levels likely increased by glacial melt or fresh water inflow. These affected stations are 1: 9466057, Popokamute, Kuskokwim River; 2: 9465374, Snag Point, Dillingham; 3: 9455963, Goose Creek, Cook Inlet; 4: 9454240, Valdez; 5: 9452346, Cove Point.**

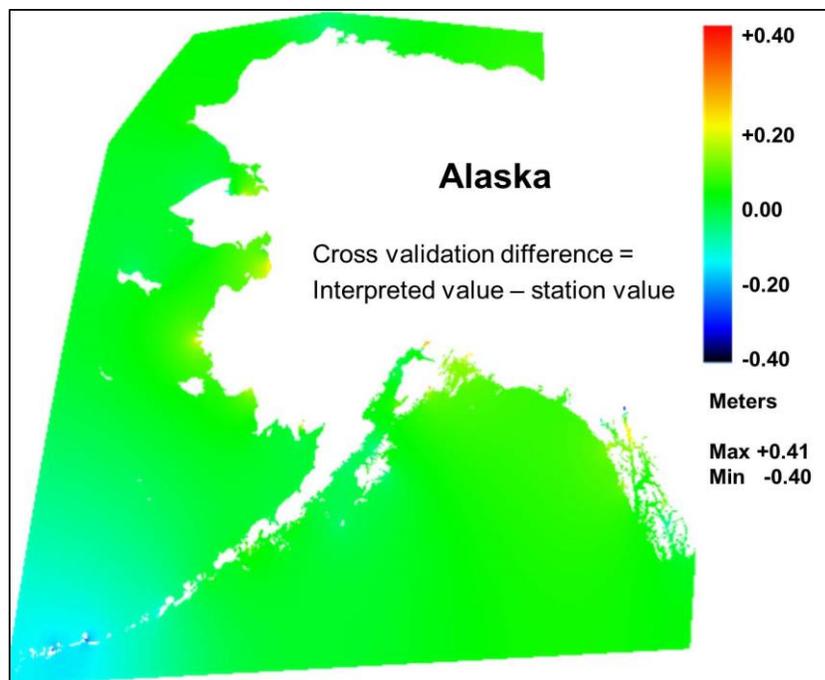
about 0.35 m above the geoid at Atka Island near the western tip of the Aleutian Island chain then increases to over 1.6 m above near Juneau in southeast Alaska; this gradient is approximately a 1.2 meter gradual rise over 2800 km horizontally. Some localized sea level increase near locations such as eastern Cook Inlet, northern Prince William Sound and Nunam Iqua could be due to the addition of riverine or glacial melt runoff into a confined channel (figs. 1, 3), however these effects will decay quickly away from these sources.

The sensitivity of the modeled TSS to individual stations is low. The cross validation differences ranged from -0.41 m to 0.40 m with an average of 0.008 m and a standard deviation of 0.12 m. The cross validation differences were interpolated using the same Laplace interpolation method used for the TSS and is shown in Figure 5. Low sensitivity of the model to individual stations suggests that the sampling frequency is sufficient to capture both the physical processes driving the TSS and any low frequency geoid errors (which are incorporated in the TSS model by this design).

Vertical movement of land due to tectonic forces or post-glacial isostatic adjustment (GIA) is a potential source of error for these sea surface measurements. Two regions where large-scale crustal motion is likely to be an issue are the Kenai Peninsula (which divides Cook Inlet and Prince Edward Sound) and the Yakutat area northwest of Juneau. These are places where shorelines are uplifting at rates in the order of 10 to 20 mm/year [e.g. 8, 9]. Upward shoreline motion is observed as an apparent decline in sea level at a local tidal station. For this study, the largest MSL error imposed by this factor will be at tidal datum stations which have a high rate of vertical shoreline movement combined with a large time gap between the measured tidal epoch and the GNSS bench mark occupation. At many of the stations where vertical crustal movement is greatest, an updated, 5-year modified tidal epoch which runs from 2007 to 2011 is used to calculate MSL. Juneau, station number 9452210, is an extreme example of shoreline uplift; at this station there is a downward historic MSL trend of -13.14 ( $\pm 0.35$ ) mm/yr [10]. As the benchmark used was surveyed by GNSS



**Figure 4: TSS above to the Geoid12B based on Laplace interpolation of tide station data. Note that the sea surface follows a southwest slope which dips toward the western Aleutian Islands.**



**Figure 5: Laplace interpolated cross-validation results; difference between the TSS value calculated and the modelled value at the station location using all other station's data.**

most recently in 2015, four years after the tidal epoch used to measure average sea level, the height of MSL at Juneau used here could be exaggerated by 5 to 8 cm. Future TSS models should include a corrector for crustal movement.

## Conclusions

Data from ~100 tidal stations spread over the Alaskan coasts show that the TSS, or the vertical difference between MSL and the Geoid12B forms a SSW-sloped wedge. The TSS dips linearly at approximately 1.2 m vertically over 2800 km horizontally. Future versions of this TSS map should be corrected for tectonic and isostatic vertical movement of shorelines; these corrections are expected to be less than ~8 cm. This model of TSS is a significant part of the ellipsoid to chart datum separation, which has been unavailable to hydrographers in this area.

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